Maximum-flow Minimum-cut and Image Segmentation

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Maximum Flow and Minimum Cut

- Max-Flow min-cut Theorem(Ford Fulkerson,1956): In any network, the value of max flow equals capacity of minimum cut.
- Nontrivial applications: bipartite matching, airline scheduling, image segmentation, etc.

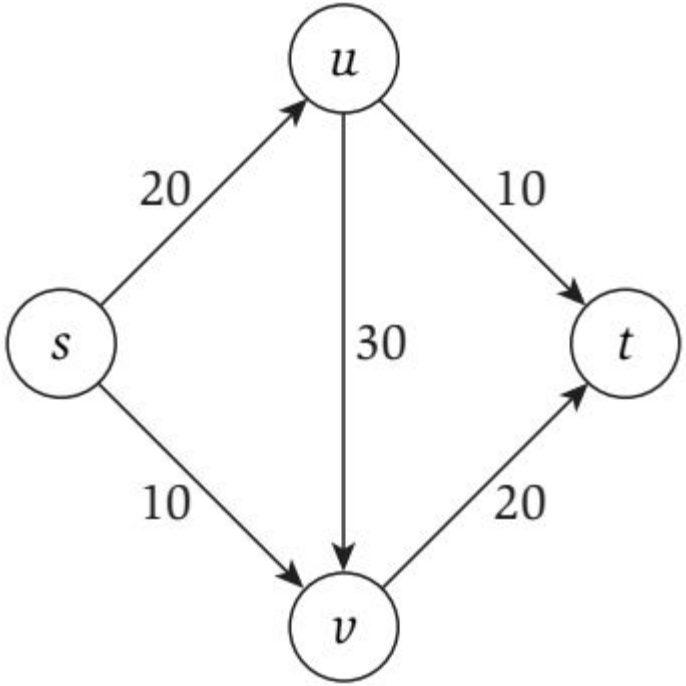


Figure 7.2 A flow network, with source s and sink t. The numbers next to the edges are the capacities.

Bipartite Matching

Find the optimal assignment from the chosen edges

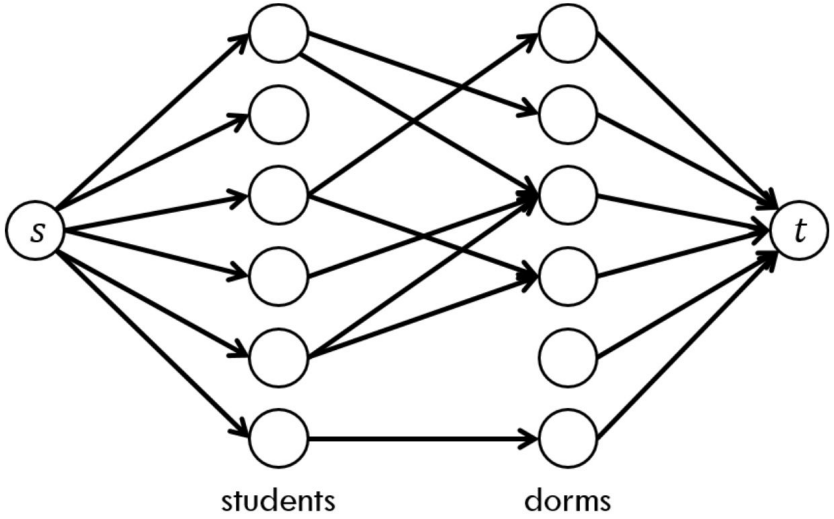


Image source: Jaehyun Park, CS97si, Stanford University

Ford-Fulkerson Algorithm

https://github.com/RuojunHong/FordFulkerson

```
Fm = 0        // initially set maximum flow = 0
While there exist an augmenting path p in G
    Find augmenting path p // p = a simple path from s to t
    Cf(p) = the smallest edge capacity on p
    Fm = Fm + Cf(p)
    for every edge on path p
        if (u,v) is an original edge, cf(u,v)=cf(u,v)-cf(p)
        else, cf(u,v) = cf(u,v) + cf(p)
        Endif
    Endfor
Endwhile
return Fm
```

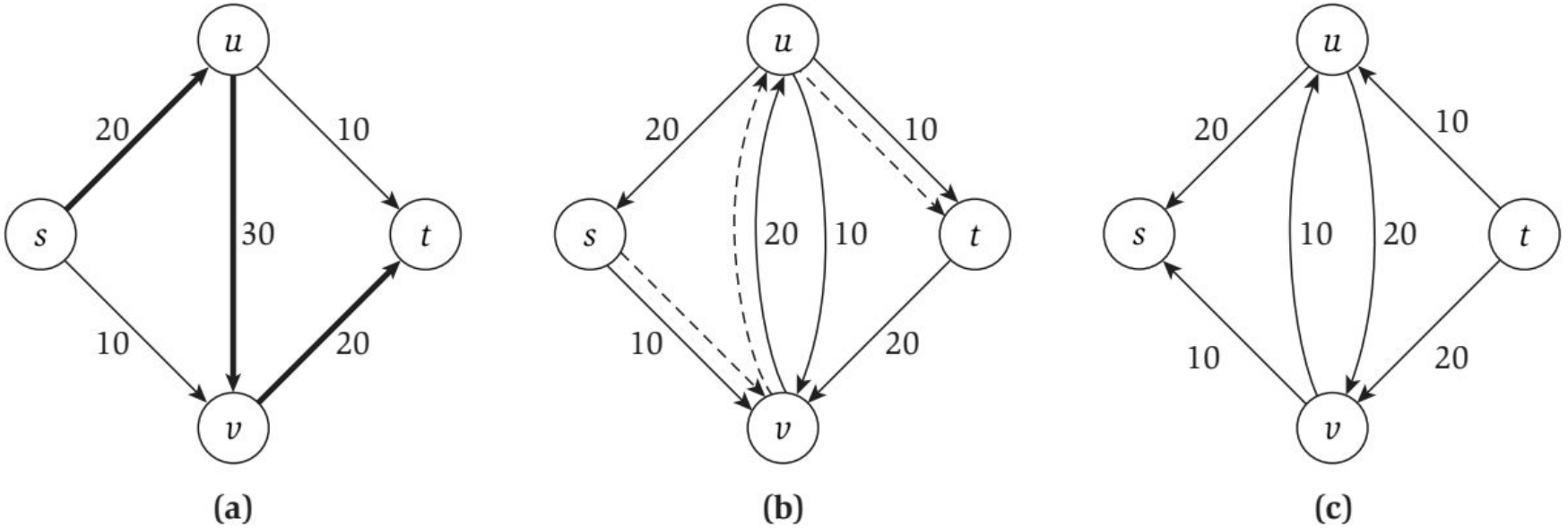


Image source: Algorithm Design, Jon Kleinberg and Eva Tardos, Tsinghua University Press (2005)

Live Demo-find maximum flow

Complexity, Augmenting Path & Graph Representation

- O(Ef)
- Image is modeled as a grid graph

- What is the best way to represent a graph?
  - adjacency matrix->ideal for grid graph
  - adjacency list
- What is the best way to find a s-t path in a graph?
  - BFS
  - DFS
  - Shortest/Maximum capacity

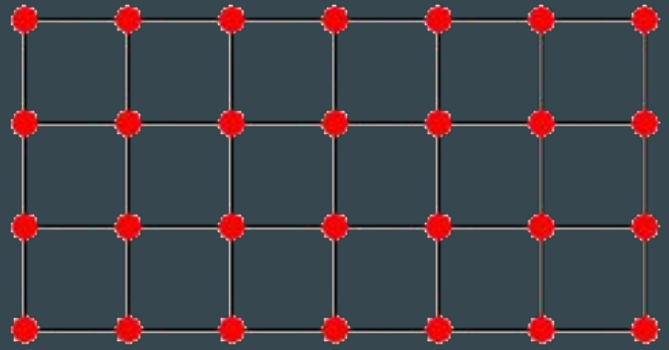


Image Segmentation

Label each pixel in a way that pixels with the same label share some common visual characteristics

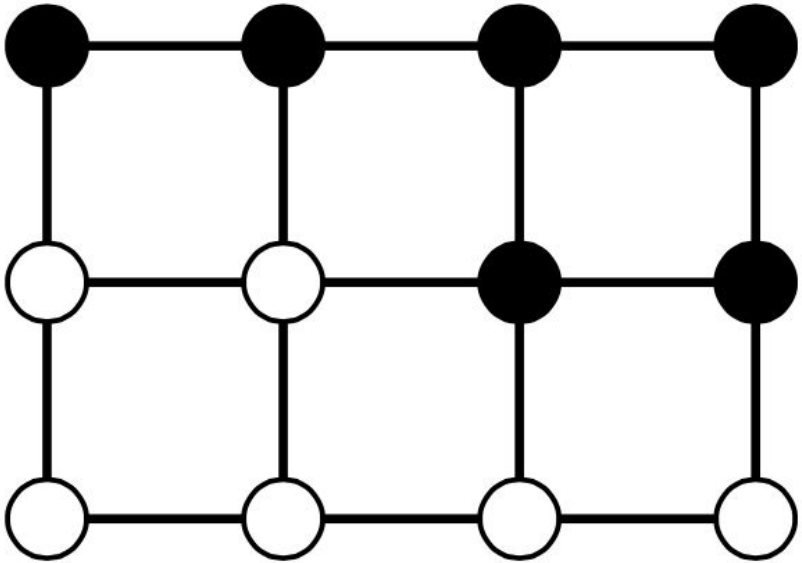


Image source: Tomas Werner, Center for Machine Perception Czech Technical University Prague

Segmentation based on the mean intensity-binary image

Minimizing Energy Function

$$E(x) = \sum_i D_i(x_i) + \sum_{i,j \in N} V_{i,j}(x_i, x_j)$$

Penalty edge capacity: λ

Data edge capacity: one being 0, the other one being the intensity of that pixel.

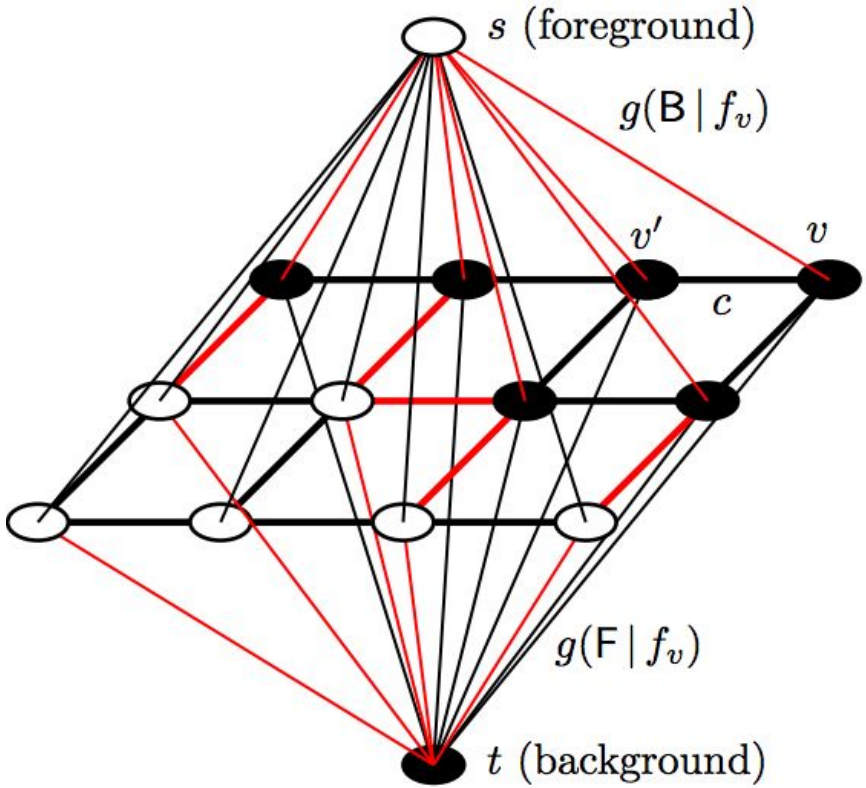


Image source: Tomas Werner, Center for Machine Perception Czech Technical University Prague

Binary Image labeling

Removing noises, isolating main objects

Original binary image

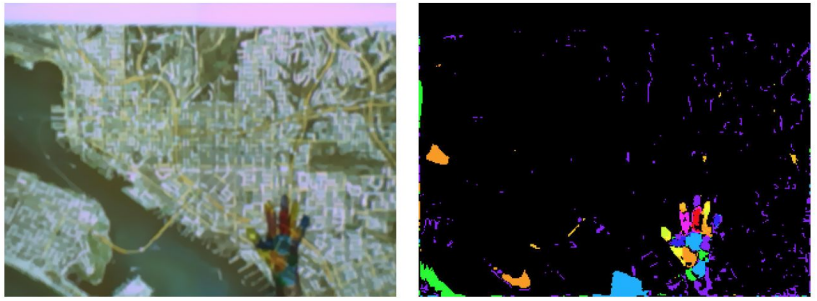
$\lambda = 100$

$\lambda = 500$

My program outputs

Binary Image Labeling

- The application of binary image restoration
- e.g. binary black and white image corrupted while being sent through a communication channel
- Can be used to track the position of the hand in camera images for gestural interaction



(a) Camera image (b) Color classified image (c) Black and white image. Image source: Ying Yin, MIT.

(a) β=0.3 (b) β=0.7 (c) β=1.1

Image source: Ying Yin, MIT.
http://people.csail.mit.edu/yingyin/resources/doc/projects/yingyin_6854_project.pdf

My replicate using my own program

λ = 300 λ = 500

Live Demo-Binary Image Pixel labeling

Extension-k-cut Labeling

- Allows k>2 labels for pixels
- NP-hard
- very important problem to be solved in computer vision

Conclusion

- Practical implementation of a theoretical approach
- First attempt in the research field
- Prepare for graduate level complexity

References

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2. Find Paths in Graphs, Robert Sedgewick, Princeton University https://www.cs.princeton.edu/~rs/talks/PathsInGraphs07.pdf
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